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Federal Communications Commission
Office of Secretary

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Mr. William F. Caton
Acting Secretary
Federal Communications Commission
1919 M Street, N.W. Room 222
Washington, D.C. 20554

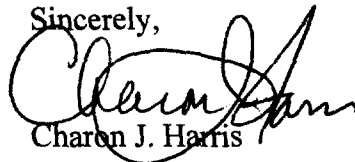
EX PARTE: Federal-State Joint Board on Universal Service
CC Docket No. 96-45

Dear Mr. Caton:

Please associate the attached statement of Professor Paul Milgrom of Stanford University with the captioned docket. As requested by FCC staff, Professor Milgrom provides this statement as the formal proof of the theoretical claims made earlier in this docket with respect to GTE's auction proposal for determining universal service support.

Please call me if you have any questions regarding this matter.

Sincerely,



Charon J. Harris

Attachment

cc: Federal State Joint Board Commissioners and Staff
A. Bush
D. Fertig
E. Kwerel
J. Morabito
G. Rosston

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Statement of Paul Milgrom

In response to the request of FCC staff, this statement supplies formal proof of the central theoretical claim made in my earlier statement on universal service auctions. The statement is designed to be read alongside Roger Myerson's paper "Optimal Auction Design," which was published in *Mathematics of Operations Research*, Vol 6, No. 1, February, 1981, pp 58-73.

Most of Myerson's analysis can be adapted to the universal service auction problem with no change in the formal analysis, provided one makes a proper reinterpretation of some of the symbols. Myerson appears to analyze an auction in which the bidders seek to acquire something of value and pay a positive price to acquire it. In contrast, the winner of the universal service auction acquires an obligation, which in itself has negative value (equal to basic service rate minus the actual cost of providing service), but the winning bidder receives compensation for the obligation in the form of support payments. As a matter of formal mathematics, however, Myerson's analysis applies to this second case as well provided we treat all the values and support payments as negative numbers. The winning bidder acquires an object of negative value and "pays" a negative price for it.

To apply Myerson's analysis to the universal service problem, I reinterpret $p_j(t)$ to be the proportion of the market served by bidder j when the list of bidder types is that given by t . (Myerson had interpreted the same symbol to be the probability that bidder j would be the winning bidder when the list of bidder types is t .) I here assume that the cost of service is proportional to the winner's market share and that market shares are equal among auction winners in a CBG.¹ With this interpretation and assumption, the formula for the bidder's expected profits from the auction are exactly those given by Myerson's formula (3.1). On account of this identity, all the derivations reported in Myerson's paper based on (3.1) apply exactly. This allows me to omit the greatest part of the detailed analysis here. The changes required to adapt Myerson's analysis to account for the different objective of the auction designer are noted below.

The first change is to formula (3.2). Myerson assumes that the auction designer is a seller who wishes to maximize its expected profit and his formula (3.2) expresses that objective. In my earlier statement, the auction designer is instead a government regulator whose objective is to maximize

Expected Benefits to Consumers
- *Expected Costs Incurred by the COLRs*
- $\alpha \times$ *Expected Support Payments to COLRs*

In mathematical terms, changing the objective is accomplished by replacing (3.2) on page 61 by:

¹ As reported in my initial statement, this assumption can be relaxed to cover the case where the bidders have equal fixed costs of service. However, in order to rely on Myerson's initial formulation, I omit the fixed cost element from this analysis.

$$\int_T \left(B(p(t)) + \sum_{j \in N} p_j(t) v_j(t) + \alpha \sum_{j \in N} x_j(t) \right) f(t) dt \quad (3.2')$$

The term involving $B(p(t))$ expresses the expected benefit enjoyed by consumers when the market shares are as listed in $p(t)$. The middle term expresses the expected costs. (The “plus” sign is not a mistake in this term or the next; the terms are negative because the $v_j(t)$ and $x_j(t)$ are negative.) The last term subtracts the burden imposed by the support payments. All these terms are multiplied by a probability density and then integrated to obtain their expected values.

The next required changes are on pages 64-65, where (4.7), (4.9), and (4.12) all involve the representation of the auction designer’s objective. These need to be replaced by the corresponding primed expressions shown below:

$$\int_T \left(B(p(t)) + \sum_{j \in N} p_j(t) \left[(1 + \alpha) v_j(t) + \alpha \frac{1 - F_i(t_i)}{f_i(t_i)} \right] \right) f(t) dt \quad (4.7')$$

$$U_0(p, x) = \int_T \left(B(p(t)) + \sum_{j \in N} p_j(t) v_j(t) + \alpha \sum_{j \in N} x_j(t) \right) f(t) dt \quad (4.9')$$

$$U_0(p, x) =$$

$$\int_T \left(B(p(t)) + \sum_{j \in N} p_j(t) \left[(1 + \alpha) v_j(t) + \alpha \frac{1 - F_i(t_i)}{f_i(t_i)} \right] \right) f(t) dt - \sum_{i \in N} U_i(p, x, a_i) \quad (4.12')$$

The upshot of (4.12') is that at the bottom of page 65, the Corollary (called the “Revenue Equivalence Theorem”) based on (4.12) can be replaced by the following italicized statements, which includes one of the more important claims made in my earlier statement:

PAYOFF EQUIVALENCE THEOREM: *The payoffs to the public from a feasible auction mechanism is completely determined by the market share function p and the numbers $U_i(p, x, a_i)$ for all i . That is, once we know what shares are awarded in each possible situation (as specified by p) and how much expected payoff each bidder would get if his cost estimate were as high as possible, the expected support payment does not depend on the payment function x . For example, comparing a uniform price auction in which each winning bidder receives the same support payment with a “discriminatory” auction in which bidders’ support payments may differ, depending on their bids, we may conclude that both lead to the same level of expected support payments, provided the identities of the winning bidders remains unchanged.*

From (4.8), we infer (exactly as Myerson does) that an optimal auction must specify that each $U_i(p, x, a_i)$ equals zero, that is, that a bidder with very high costs expects to earn a zero profit. This condition is satisfied for new potential COLRs in my proposed auction design, because bidders with very high costs would be unable to bid less than the reserve. For ILECs, this is still the case if the reserve is set appropriately based on a cost model.

The analysis in my previous statement was limited to the regular case, which is the case in which $c_i(t_i)$ is strictly increasing (and so in which the constraint (4.2) is not binding). In that case, for any types t , the optimal auction selects $p(t)$ to maximize the integrand in (4.7'). That is equivalent to maximizing the following for each type vector t :

$$\frac{1}{1+\alpha} B(p(t)) + \sum_{j \in N} p_j(t) \left(v_j(t) + \frac{\alpha}{1+\alpha} \frac{1 - F_j(t_j)}{f_j(t_j)} \right) \quad (*)$$

This completes the formal part of the proof. The rule for identifying the winning bidders is the same as described in my earlier statement, but I have reorganized the terms here to give it a slightly simpler form. The key conclusion is that in an optimal auction, the winners in each set of circumstances are determined by a weighted comparison of the benefits to consumers against a cost term that is adjusted to account for bidding incentives. As the weight α accorded to minimizing the universal service fund increases, the auction attaches smaller relative weight to the benefits of competition and greater relative weight to keeping bidder profits small.